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Spherical Gravitational Waves in Relativistic Theory of Gravitation

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Within the framework of relativistic theory of gravitation the exact spherically-symmetric wave solution is received. It is shown that this solution possesses the positive-definite energy and momentum deriving with the Fock energy-momentum density tensor of gravitational field. In this connection the sense of Birkhoff theorem in Relativistic Theory of Gravitation is discussed.

I. SPHERICALLY-SYMMETRIC WAVE SOLUTION

In General Relativity the energy of gravitational field is described with help the energy-momentum pseudotensor, but the expression for corresponding tensor is absent. It is one of reasons for regarding the gravitation interaction as a tensor interaction in Minkowski space-time (see, for example [1]). The most consistently such approach was realized in the so-called relativistic theory of gravitation (RTG) [2], [3]. This theory can be considered as a gauge theory of the group of Lie variations for dynamical variables. The related transformations are variations of the form of the function for generally covariant transformations. In order to the action be invariant for this group under the transformations of the dynamic variables alone one needs replacing the "nondynamic" Minkowski metric γ^{ik} with expression g^{ik}: $\tilde{g}^{ik} = \sqrt{-g}g^{ik} = \sqrt{-\gamma}(\gamma^{ik} + k\psi^{ik})$, where $\gamma = det\gamma_{ik}, g = detg_{ik}$, k^2 - is the Einstein constant, and thus introducing the gauge gravitational potential ψ^{ik}. The quantity g^{ik} is interpreted here as a metric of an effective space-time allowing an unique construction of the connection (the Cristoffel bracketS). The RTG field equations at its massless variant are the Einstein ones for this effective metric, added the conditions, restricting the spin states of the field ψ^{ik}

$$D_i\tilde{g}^{ik} = 0 \ , \tag{1}$$

where D_i - the covariant derivative in the Minkowski space. This condition plays the significant role in RTG. It removes the gauge arbitrariness of Einstein equations and coincides with the Fock harmonical condition in Galilean coordinates [4].

Although the massless RTG field equations locally coincide with General Relativity ones, their global solutions, generally speaking, will be different, since this solutions are defined on the various manifolds. RTG, being founded on the simple space-time topology, allows to introduce the global Galilean coordinate system, that distinguishes RTG from the bimetric theories, in which a flat space plays the auxiliary role and its topology does not define the character of the physical processes. This distinction may take place at interpretation of the field solutions, since the coordinate system in RTG is defined by Minkowski metric,

but it is fixed by noncovariant coordinate conditions in GR. Just this situations takes place for spherical-symmetric gravitational fields. In GR according the Birkhoff theorem [5] any spherical gravitational field in vacuum is static. The proof of this theorem is grounded on the transformation of certain spherically-symmetric metric to the coordinates in which it has a static form. But in RTG such transformation is the transfer from the spherical coordinates in the Minkowski space with the metric $\gamma_{ik} = diag(1, -1, -r^2, -r^2 \sin^2\theta)$ to some "nonstatic" coordinates. The Birkhoff theorem means that in the case of spherical symmetry the coordinate system in which the vacuum metric depends from one coordinate always exists, but it not means that the field was static in the starting coordinates.

Hence the task of the investigation of nonstatic spherical-symmetric solutions, which was in general view investigated in [6], arises. In this paper one of the possible nonstatic spherically-symmetric wave solution is found in implicit form. Such solutions may play very important role in astrophysics and cosmology.

To find the spherical wave solutions we use the Birkhoff theorem and present a non-static spherical vacuum solution in the certain coordinate system (τ, R, θ, ϕ) in the form of Schwarzschild metric

$$ds^2 = (1 - \frac{2m}{R})d\tau^2 - (1 - \frac{2m}{R})^{-1}dR^2 - R^2 d\Omega^2 \ . \tag{2}$$

To find the solution in spherical coordinates of (t, r, θ, ϕ) we make the coordinate transformation

$$t = t(\tau, R), r = r(\tau, R) \ , \tag{3}$$

The transformation coefficients will be found from the condition (1). The corresponding equations connecting the variables (t, r) and (τ, R), take the form

$$\frac{R}{R-2m}\frac{\partial^2 t}{\partial \tau^2} - R^{-2}\partial_R[(R^2 - 2mR)\frac{\partial t}{\partial R}] = 0 \ , \tag{4}$$

$$\frac{R}{R-2m}\frac{\partial^2 r}{\partial \tau^2} - R^{-2}\partial_R[(R^2 - 2mR)\frac{\partial r}{\partial R}] + \frac{2r}{R^2} = 0 \ . \tag{5}$$

We search for the partial solution of the equations (4), (5) in the next form

$$\tau = t + T(u), R = r + m, u = t + f(r) \ , \tag{6}$$

where u - the retarded argument, which is finite at any values of r; the light velocity and gravitational constant are believed equal to unit. Another form of solution was considered in [7]. Finding with help of (6) the transformations coefficients and substituting its to equations (4),(5), we find that the equation (5) is satisfied identically, and the equation (4) after variables separation is reduced to ordinary differential equations with separation constant C for the functions $T(u)$ and $f(r)$

$$T_{uu} = CT_u(1 + T_u)^2 \ , \tag{7}$$

$$f_{rr} + C{f_r}^2 + \frac{2r}{r^2 - m^2}f_r - C\frac{(r+m)^2}{(r-m)^2} = 0 \ , \tag{8}$$

The first integral of equation (7) has a form

$$Cu = \frac{1}{1+z} - \ln|\frac{z+1}{z}| + A \ , \tag{9}$$

where $z = T_u$, A - integration constant. From (9) we find that function $z(u)$ is positive for positive A and large values of u $(u > 0)$.

The equation (8) may be integrated with help the power series for $f(r)$ relative variable $1/r$. The analysis of this solution shows that for $C > 0$, $f_r < 0$ for all values of r. The metric components now have the next form

$$g_{00} = \frac{r-m}{r+m}(1+T_u)^2 \ , g_{01} = \frac{r-m}{r+m}T_u(1+T_u)f_r \ , \tag{10}$$

$$g_{11} = \frac{r-m}{r+m}{T_u}^2{f_r}^2 - \frac{r+m}{r-m} \ , g_{22} = -(r+m)^2 \ , g_{33} = -(r+m)^2\sin^2\theta \ . \tag{11}$$

II. FOCK ENERGY-MOMENTUM TENSOR OF GRAVITATIONAL FIELD

The finding metric may be used for the investigation of a sign of gravitational radiate density t^{00}, which according to RTG has a form [2]

$$t^{00} = \frac{1}{\sqrt{-\gamma}}\gamma^{ik}D_iD_k\tilde{g}^{00} \ . \tag{12}$$

The problem concerning positive definite of gravitational energy density is not trivial, because in RTG the expression t^{00} don't possess square-law structure relative the field functions and its first derivatives and it contains second derivatives too. This situation is the consequence of a difference between spin one and spin two description. In the case of a vector field Lagrangian does not contain the Christoffel symbols (in arbitrary coordinate system) and the energy-momentum tensor can be easily found when this Lagrangian is varied in respect to the metric. The same is true for the Maxwell and Yang-Mills fields too. Quiet different situation arises in the case of a tensor field. Now the Christoffel symbols can not be shorten and thus the energy-momentum tensor contains the second derivatives. This fact leads, generally speaking to uncertainty of energy density sign. The possible approaches to the solving of this problem are regarded in [8],[9].

Let us consider Fock representation for Einstein tensor G^{ik} [4]

$$2gG^{ik} = \partial_m\partial_n(\tilde{g}^{ik}\tilde{g}^{mn} - \tilde{g}^{im}\tilde{g}^{kn}) + L^{ik} \ , \tag{13}$$

where the term L^{ik} contain the first derivatives only. The expression

$$U^{ik} = \partial_m\partial_n(\tilde{g}^{ik}\tilde{g}^{mn} - \tilde{g}^{im}\tilde{g}^{kn}) \ , \tag{14}$$

Fock interpreted as gravitational density energy-momentum pseudo-tensor of weight equal to +2. Due to field equations it equal to L^{ik} and coincides with well-known Landau-Lifshitz pseudo-tensor.

In RTG we may to receive the corresponding tensor density, replacing ordinary derivations to covariant derivations relative Minkowski metric γ^{ik}

$$T_g^{ik} = \frac{1}{\sqrt{-\gamma}}D_mD_n(\tilde{g}^{ik}\tilde{g}^{mn} - \tilde{g}^{im}\tilde{g}^{kn}) \ , \tag{15}$$

This tensor may be received by means of variational procedure from the next Lagrangian

$$L = \tilde{R}(g^{ik}) + \frac{1}{\sqrt{-f}} R_{ijkl}(f_{mn}) \tilde{g}^{ik} \tilde{g}^{jl} \ . \tag{16}$$

where $\tilde{g}^{ik} = \tilde{f}^{ik} + k\tilde{\psi}^{ik}$, f_{ik} is background metric with nonzero curvature tensor, which is believed equal to zero after variation. Analogously approach was used in [10]to receive the energy-momentum tensor of conformal-invariant scalar field.

Using Fock energy-momentum tensor, let us to calculate the energy and momentum density for finding solution. For energy density we have

$$T_g^{00} = -\frac{1}{r^2}[(\frac{R^4}{r^2})_r + 2\frac{R^2}{r}({\tau_r}^2 g_{\tau\tau} + {R_r}^2 g_{RR})]_r \ , \tag{17}$$

This expression is the sum of static and wave parts. For later we have

$$T_w^{00} = -\frac{2}{r^2}[(\frac{R^2}{r} g_{\tau\tau})_r {T_u}^2 {f_r}^2 + 2\frac{R^2}{r} g_{\tau\tau}(T_u T_{uu} {f_r}^3 + {T_u}^2 f_r f_{rr})] \ . \tag{18}$$

The expression (17) may be present as

$$T_w^{00} = \frac{2{T_u}^2}{r^2}[(3 - \frac{m^2}{r^2}){f_r}^2 - 2Cf_r r(1 - \frac{m^2}{r^2})(\frac{(r+m)^2}{(r-m)^2} + {f_r}^2({T_u}^2 + 2T_u))] \ . \tag{19}$$

The analysis of last expression shows that it is a positive-definite, if C and T_u are positive, but f_r is negative. As it was finding above this conditions follow from solutions of equations (7),(8).

Let us to find the expression for momentum density.In Cartesian coordinates x^i we have

$$T_w^{0i} = \frac{4Cx^i}{r^4}(r^2 - m^2){f_r}^2 {T_u}^2 (1 + T_u)^2 \ , \tag{20}$$

This expression is positive-definite for $C > 0$ that testify about energy transport and, consequently, about a wave character of finding solution.

For large values of r we have

$$f_r = -1 - \frac{1}{Cr} \ , \tag{21}$$

$$T_w^{00} = \frac{4C}{r}{T_u}^2(1 + T_u)^2 \ , \tag{22}$$

$$T_w^{0i} = \frac{4Cx^i}{r^2}{T_u}^2(1 + T_u)^2 \ . \tag{23}$$

The expressions (22),(23) show that energy and momentum are not transporting at infinity as it had to be, because wave solution was finding with help the coordinate transformation from asymptotically-flat metric.

Note that the asymptotical expression for retarded argument coincides with corresponding expression, using by Fock [4] in wave zone

$$u = t - (r + 2m \ln \frac{r}{r_0}) \ , \tag{24}$$

whence follow that $C = 1/2m$.

So, we may to conclude that Hilbert energy-momentum tensor play the role of gravitational current in the field equations although Fock tensor describes the energy characteristics of gravitational field.

III. CONCLUSION

The existence of spherically-symmetrical wave solutions and as a whole nonstatic spherically-symmetrical solutions is in a contradiction with ordinary physical interpretation of Birkhoff theorem in GR. In RTG such solutions have a physical sense as far as the temporal coordinate of the Minkowski space-time has it. Essentially that this wave solution possesses positive-defined energy and momentum densities.

Although the receiving solution has enough formal character, it illustrates the possibility of existence of spherical gravitational waves. A total solution must include in the interior one and the matching of these solutions.

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